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The Solutions of Non-linear Wave Equations by Variational Iteration Method

Rabeea Mohammed Hani Darghoth
University of Mosul, College of Basic Education, Mathematics Department, Mosul, Iraq

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Correspondence:
Rabee Mohammed Hani
Email: rabeeamo@uomosul.edu.iq

Abstract

Abstract: This paper utilizes the Variational Iteration Method to address non-linear wave equations. This study provides approximate analytical solutions for both homogeneous and non-homogeneous non-linear wave problems. Moreover, the study provided specific analytical solution formulas for specific cases, which allowed for the determination of the analytical solution without the need for successive iterative operations. Furthermore, this study provided a range of various solved examples and compared the approximate solution with the analytical solution.

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حلول معادلات الموجة غير الخطية باستخدام طريقة التكرار التبايني

ربيع محمد هاني

جامعة الموصل، كلية التربية الأساسية، قسم الرياضيات، الموصل، العراق

المستخلص

توظف هذه الدراسة طريقة التكرار التبايني لمعالجة معادلات الموجة الغير الخطية، بهدف الحصول على حلول تحليلية تقريبية لكل من المسائل المتجانسة وغير المتجانسة لمعادلات الموجة غير الخطية. كما تتضمن الدراسة اشتقاق صيغ تحليلية دقيقة لحالات خاصة محددة، الأمر الذي من الممكن التوصل إلى الحل التحليلي المباشر دون الحاجة إلى عمليات التكرار المتعاقبة. وتشتمل الدراسة كذلك على مجموعة متنوعة من الأمثلة التطبيقية مع حلولها، تمت من خلالها مقارنة النتائج التقريبية بالحلول التحليلية الدقيقة، بما يعزز من فاعلية صيغ الحل المقترحة وكفاءتها في معالجة هذا النوع من المعادلات.

كلمات مفتاحية: الحل التحليلي التقريبي، معادلات الموجة الغير الخطية، طريقة التكرار التبايني.

1. Introduction

The iterative methods are semi-analytical techniques that address different types of differential equations. These techniques are advantageous, especially for non-linear partial differential equations that are challenging to solve using conventional analytical methods. Several recent studies have employed these iterative techniques, including the Darghoth study (Darghoth, 2024), which derived an approximate solution to the nonlinear Harry–Dym equation through successive iterations. The Mohammed study (Mohammed, Al-Ramadhani, & Darghoth, 2022) achieved an analytical solution to the nonlinear KdV-type partial differential equation through iteration.

The Variational Iteration method (VIM) is a significant iterative technique, noted for its robustness and efficiency, and it does not necessitate intricate analyses. Recently, it has been widely employed to solve non-linear partial differential equations.

He's study (He, Variational iteration method--a kind of non-linear analytical technique: some examples, 1999) pioneered this methodology, facilitating the resolution of numerous intricate and formidable issues.

This study investigated various issues related to this technique, demonstrating that it does not require a small parameter in an equation, in contrast to other methods. It calculated the semi-analytic solution for various non-linear initial problems. Wazwaz (Wazwaz, The variational iteration method for solving linear and nonlinear ODEs and scientific models with variable coefficients, 2014) utilizes this technique to resolve various ODEs models. It has been used to solve a variety of partial differential equations, including, Pamuk (Pamuk, 2022) solve the linear and non-linear heat equation via this technique.

In this study, we consider the general form of non-linear wave equation in one dimension:

$$\frac{\partial^2 u}{\partial t^2} - \frac{\partial}{\partial x}\left(\omega(u).\frac{\partial u}{\partial x}\right) = S(x,t), \tag{1}$$

with initial conditions

$$\begin{aligned} u(x,0) &= f(x), \\ u_t(x,0) &= g(x) \end{aligned} \tag{2}$$

Here, the indices t and x denote the derivatives with respect to these variables, the functions S, ω are the source function, , speed of sound respectively, noting that the function S depends on the solution u or

constant, and f and g are any differentiable functions ,The nonlinear wave equation represents one of the most fundamental categories of nonlinear hyperbolic equations (Hunter, 1996).
Hemeda (Hemeda, 2008) used this method to solve various forms of wave equations, and he obtained an exact solution; however, these forms were either the first order or the speed of sound parameter is a constant number, these formulas are distinct from equation (1) that is going to addressed in this study.
Linear wave equations such as wave-like equations are studied, some in bounded and others in unbounded domains, by Wazwaz (Wazwaz, The variational iteration method: A reliable analytic tool for solving linear and nonlinear wave equations, 2007), He also studies wave equations of the form:

$$u_{tt} - u_{xx} = F(u) + S(x,t)$$

where the function S is referred to as the source function. The He' study (He & Wu, Variational iteration method: New development and applications, 2007) presented an improved iterative strategy and offered a new iterative formulation to overcome the shortcomings of traditional Variational methods for solving nonlinear wave equations.

$$u_{tt} - c^2 u_{xx} - F(u, u_t, u_{tt}, u_x, u_{xx}, u_{xt}, u_{xxx}, \ldots) = 0$$

Both studies differ from eq. (1),which we will be addressing in this work.

2. The variational iteration method

Consider the differential equation:

$$Lu + Nu = S(x,t), \tag{3}$$

where L and N are linear and nonlinear operators respectively, the variational iteration method presents a correction functional for equation in the form:

$$u_{n+1} = u_n + \int_0^t \lambda (Lu_n + N\tilde{u}_n - S) d\xi \,, \tag{4}$$

Where λ is a general Lagrange's multiplier. $\tilde{u}_n$ is a restricted value that means it behaves as a constant. $\delta\tilde{u}_n = 0$, where δ is the variational derivative. (Wazwaz, Linear and Nonlinear Integral Equations , 2011)

Taking the variation of the equation (4) with respect to the independent variable u_n and multiplying both sides by δu_n to obtain

$$\delta u_{n+1} = \delta u_n + \delta \int_0^t \lambda(\xi) L u_n(x,\xi) d\xi, \tag{5}$$

here

$$\frac{\delta}{\delta u_n}\big(\lambda(\xi)(N\tilde{u}_n(x,\xi) - S(x,\xi))\big) = 0.$$

For applying equation (4) effectively, it requires identifying the linear terms of equation (1), consequently, if $\omega(u) = C$, where C is non-negative real number, this constant coefficient indicates of the wave's speed of sound., then the linear terms of eq. (1) are $Lu_n = \frac{\partial^2 u}{\partial t^2} - C\frac{\partial^2 u}{\partial x^2}$ therefore, the VIM requires the extremum condition $u_{n+1} = 0$, then the eq. (5) becomes

$$\delta u_n + \delta \int_0^t \lambda(\xi)\frac{\partial^2 u}{\partial \xi^2} d\xi - \delta \int_0^t \lambda(\xi) C \frac{\partial^2 u}{\partial x^2} d\xi = 0 \tag{6}$$

The second term of eq. (6) will the Integrated by parts twice, and since $\left.\frac{\partial u_n}{\partial t}\right|_{t=0} = u_n(0,x) = 0$, consequently

$$\begin{aligned}\big(1 - \lambda'|_{\xi=t}\big)\delta u_n + \delta\lambda|_{\xi=t}\frac{\partial u_n}{\partial \xi} + \delta \int_0^t \lambda''(\xi)\, u_n(x,\xi) d\xi \\ - \delta C \int_0^t \lambda(\xi)\frac{\partial^2 u}{\partial x^2} d\xi = 0\end{aligned} \tag{7}$$

The stationary conditions mandate that all coefficients of the function and its derivatives must equal zero (He, Variational iteration method--a kind of non-linear analytical technique: some examples, 1999), (Hemeda, 2008), (Pamuk, 2022), (Wazwaz, The variational iteration method for solving linear and nonlinear ODEs and scientific models with variable coefficients, 2014) , leading to the following result:

$$1-\lambda'|_{\xi=t}=0\,,\ \lambda(t)=0 \text{ and } \lambda''|_{\xi=t}=0. \tag{8}$$

This in turn gives

$$\lambda(\xi)=\xi-t \tag{9}$$

The correction functional eq. (4) for equation eq.(1) becomes:

$$u_{n+1}=u_n+\int_0^t (x-\xi)\left(\frac{\partial^2 u_n}{\partial \xi^2}-C\frac{\partial^2 u_n}{\partial x^2}-S(x,\xi)\right)d\xi, \tag{10}$$

Now, consider a more general instance when $\omega(u)$ is any real function, then the only linear term of eq. (1) is

$$Lu_n=\frac{\partial^2 u}{\partial t^2}$$

This gives same Lagrange multiplier in eq. (9), and the correction functional eq. (4) for equation eq.(1) takes the form:

$$u_{n+1}=u_n+\int_0^t \lambda(\xi)\left(\frac{\partial^2 u_n}{\partial \xi^2}-\omega\frac{\partial^2 u_n}{\partial x^2}-\frac{\partial \omega}{\partial x}\frac{\partial u_n}{\partial x}-S(x,\xi)\right)d\xi\,, \tag{11}$$

The successive approximations $u_{n+1}, n>0$, of the solution $u(x,t)$ will be readily obtained upon using selective function $u_0(x,t)$. However, for fast convergence, the function $u_0(x,t)$ should be selected by using the initial conditions as follows:

$$u_0(t,x)=f(x)+\ tg(x). \tag{12}$$

Consequently, the solution of eq. (1) is

$$u(x,t)\ =\ \lim_{n\to\infty} u_n(x,t) \tag{13}$$

In other words, the correction functional will give several approximations, therefore, the exact solution is obtained as the limit of the resulting successive approximations.

3. Specific Cases

There are specific cases of VIM for solving eq. (1) depending on its homogeneity, linearity, and the determination of initial condition functions, which can be shown as follows:

Case (1)

- eq. (1) is a homogenous equation.
- $\omega(u) = C^2$, where C is a non-negative real constant.
- $g(x) = -Cf'(x)$.

Here f is a general function of a single variable that characterizes the waveform of a wave propagating at a velocity along the x-axis. (Kiselev, 2007), the eq. (1) and eq. (2) under this case will be the classical wave equation in one dimension,

$$\frac{\partial^2 u}{\partial t^2} - C^2 \frac{\partial^2 u}{\partial x^2} = 0, \tag{14}$$

with initial conditions

$$u(x,0) = f(x),$$
$$u_t(x,0) = -Cf'(x) \tag{15}$$

and via the iteration eq. (4) with general Lagrange's multiplier λ that derived in eq. (9), applying it several times until the nth term is obtained as follows:

$$u_n = f(x) - Ctf'(x) + \frac{1}{2}C^2t^2f''(x) - \frac{1}{6}t^3C^3f'''(x) + \cdots + \frac{1}{n!}t^nC^nf^n(x). \tag{16}$$

By using of eq. (13), the exact analytical solution of eq. (1) is

$$u(x,t) = f(x - Ct) \tag{17}$$

This solution eq. (17) has been mentioned by Kiselev (Kiselev, 2007).

Case (2)

- eq. (1) is a homogenous equation.
- $\omega(u) = C$, where C is a real constant.
- f and g are linear polynomial functions.

The eq. (1) and eq. (2) under this case will be as

$$\frac{\partial^2 u}{\partial t^2} - C^2 \frac{\partial^2 u}{\partial x^2} = 0, \tag{18}$$

with initial conditions

$$u(x,0) = ax \mp b, \\ u_t(x,0) = cx \mp d \tag{19}$$

The $a, b, c,$ and d are real numbers. The initial conditions will lead to the first iteration shown in eq.(12), since the second derivative with respect to time and location is zero. This means that all repetitions are equal, as shown in eq. (10) and eq. (13). This means that the analytical form solution of eq. (1) is

$$u(x,t) \ = \ f(x) + \ tg(x) \tag{20}$$

Case (3)

- eq. (1) is a homogenous equation.
- $\omega(u) = \frac{1}{u}$ is a Multiplicative inverse function.
- $g(x) = Af(x)$. where A is a non-zero constant.
- $f^m(x) = A^m f(x), m = 0,1,2.$

These conditions resulted in the disappearance of the non-linear terms in eq. (1) this means, at any iteration, we have $N\tilde{u}_n = 0$. Then, the eq. (1) is

$$\frac{\partial^2 u}{\partial t^2} - \frac{1}{u}\frac{\partial^2 u}{\partial x^2} + \frac{1}{u^2}\left(\frac{\partial u}{\partial x}\right)^2 = 0, \tag{21}$$

the successive approximations $u_n, \forall n > 0$ via eq.(11) gives us follows:

$$u_n = (At + B)f(x) + 0 + 0 + 0 + \cdots$$

Using the eq. (13), the specific analytical solution to this problem is:

$$u(x,t) = (1 + At)f(x). \tag{22}$$

Case (4)

- $f(x) = -f''(x), \ g(x) = 0$,
- $\omega(u) = 1,$
- $S(x,t) = u(x,t) = f(x) + h(t).$

Since $S = u$, the eq. (1) and eq. (2) will be

$$\frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial x^2} = u, \tag{23}$$

$$u(x,0) = f(x) \, , u_t(x,0) = 0 \tag{24}$$

therefore the eq. (23) and eq.(24) with these above conditions can be referred to as the Klein Gordon equation in one dimension, see Polyanin (De Jager, 1967), hance the linear parts of eq. (23) are

$$Lu_n = \frac{\partial^2 u}{\partial t^2} - u \tag{25}$$

Consequently, we should reformulate eq. (8) as

$$1 - \lambda'|_{\xi=t} = 0\,,\ \lambda(t) = 0 \ \text{ and } \ \lambda''|_{\xi=t} - \ \lambda = 0. \tag{26}$$

So that the λ function may be determined by solving this equation to obtain

$$\lambda(\xi) = \sinh(\xi - t) \tag{27}$$

This matches Yousif's work (Yousif & Mahmood, 2017), in which he employed the homotopy perturbation approach to solve the Klein–Gordon and sine-Gordon equations. The first iteration is $u_0(t,x) = f(x)$ and employing equations (11) with eq. (27), the nth approximation solution as the followings:

$$u_n(x,t) = f(x) + \frac{t^2}{2!} + \frac{t^4}{4!} + \frac{t^6}{6!} + \frac{t^8}{8!} + \cdots + \frac{t^{2n}}{(2n)!}. \tag{28}$$

By eq. (13), the analytic form solution of this case is

$$u(x,t) = f(x) + \cosh(t). \tag{29}$$

Case (5)

- $\frac{\partial}{\partial x}\left(\omega(u_0).\frac{\partial u_0}{\partial x}\right) = S(x,t),$

here $u_0(t,x) = f(x) + g(x)t$, applying this condition will result in the integral term of eq.(11) vanishing. Applying this strategy yields the following nth approximation solution:

$$u_n(x,t) = f(x) + g(x)t + 0 + 0 + 0 + \cdots. \tag{30}$$

By eq. (13), we can obtain the following analytical solution:

$$u(x,t) = f(x) + g(x)t. \tag{31}$$

4. Applications for VIM

Problem I

Let

$$f(x) = \sin(x),$$
$$g(x) = -2\cos(x)\,, \tag{32}$$

$$\omega(u) = 4, \qquad S(x,t) = 0. \tag{33}$$

All conditions of **Case (1)** are satisfied, and consequently, the exact solution derived from eq. (17) is as follows:

$$u(x,t) = \sin(x - 2t) \tag{34}$$

Problem II

The eq. (1) and eq. (2) will be solved with initial conditions

$$f(x) = Ax + D, \qquad g(x) = B\,, \tag{35}$$

and the conditions

$$\omega(u) = D, \qquad S(x,t) = 0. \tag{36}$$

Where A, B, D and C are arbitrary set constants. The eq. (1) and eq. (2) with conditions (35) and (36) will be the non-linear wave equation in one dimension, according to **Case (2)**, the analytic solution via eq.(20) as the followings:

$$u(x,t) = Ax + C + tB \tag{37}$$

This solution eq.(37) agrees with the solution mentioned in Polyanin (Polyanin & Zaitsev, 2003, p. 450) .

Problem III

We will solve the equations (28) under the following conditions:

$$f(x) = Ae^{cx}, \qquad g(x) = Be^{cx}\,, \tag{38}$$

Where A, B and C are arbitrary constants. The eq. (21) with conditions (38) will be the non-linear wave equation in one dimension; all requirements in **Case (3)** are evidently met; thus, according to the eq. (22), the particular analytical solution to this problem is

$$u(x,t) = (A + Bt)e^{cx}. \tag{39}$$

This solution eq. (39) decides with the solution that revealed in Polyanin (p. 451).

Problem IV

We proceed to solve eq.(1) and eq. (2) under the following conditions:

$$\begin{aligned} f(x) &= \ln(x^2 + x + 1), \\ g(x) &= -2, \end{aligned} \tag{40}$$

$$\omega(u) = e^{u(x,t)}, \qquad S(x,t) = 0. \tag{41}$$

The non-linear wave equation in one dimension in order two is formed by combining the eq. (1) and eq. (2) with conditions (40) and (41). We have calculated u_n by applying the procedures that were stated before, through eq. (11) yields:

$$u_n(x,t) = \ln(x^2 + x + 1) - 2\left(t - \frac{t^2}{2} + \frac{t^3}{3} - \frac{t^4}{4} + \cdots + \frac{t^n}{n}\right) \tag{42}$$

Under eq. (13), the exact solution is

$$u(x,t) = \ln(x^2 + x + 1) - 2\ln(t + 1) \tag{43}$$

This solution matches the solution of traveling with exponential nonlinearity that mentioned in Griffiths (Griffiths, G. W., & Schiesser, W. E., 2009) and Zaitsev (Zaitsev & Polyanin, 2004, p. 223).

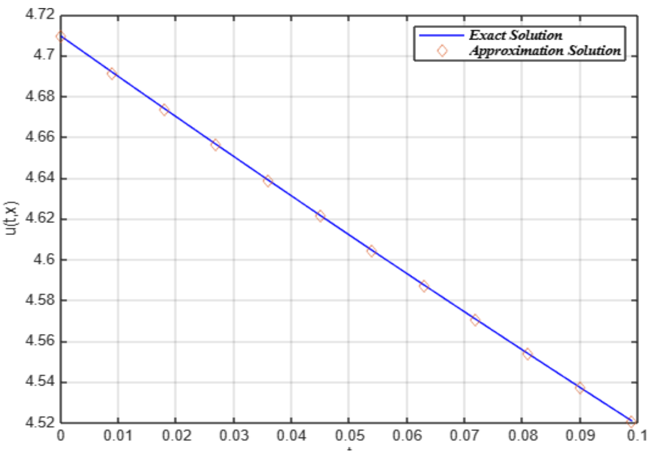

Figure 1

The Graph of eq. (43) and eq. (42) when $n = 20, x = 10$.

Problem V

Let us consider the eq. (23) and eq. (24) following conditions.

$$f(x) = \sin(x) + 1. \tag{44}$$

According to **Case (4)** , the exact solution of this problem is

$$u(x,t) = \sin(x) + \cosh(t). \tag{45}$$

and this solution eq. (45) is corresponding to the solution of the Klein Gordon equation that was provided in Yousif (Yousif & Mahmood, 2017) and El-Sayed (El-Sayed, 2003).

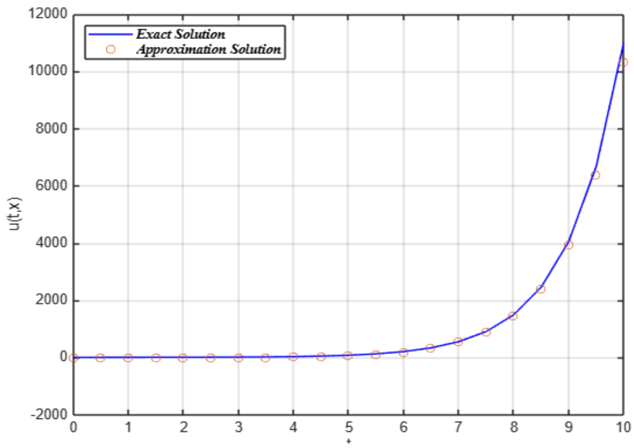

Figure 2

The Graph of eq. (45)and eq. (28) when $n = 20$, $x = \frac{\pi}{2}$

Problem VI

Let

$$f(x) = x, \qquad g(x) = 1\,, \tag{46}$$

$$\omega(u) = u, \qquad S(x,t) = 1 - x. \tag{47}$$

The eq. (1) and eq. (2) with conditions (47) is the inhomogeneous nonlinear wave equation in one dimension, the nth iteration is the following:

$$u_n(x,t) = x + t + \frac{t^3}{3!} - \frac{t^5}{5!} + \frac{t^7}{7!} + \frac{t^9}{9!} + \cdots + \frac{t^{2n+1}}{(2n+1)!}. \tag{48}$$

Using eq. (13), the analytical solution to this problem becomes

$$u(x,t) = x + \sinh(t) \tag{49}$$

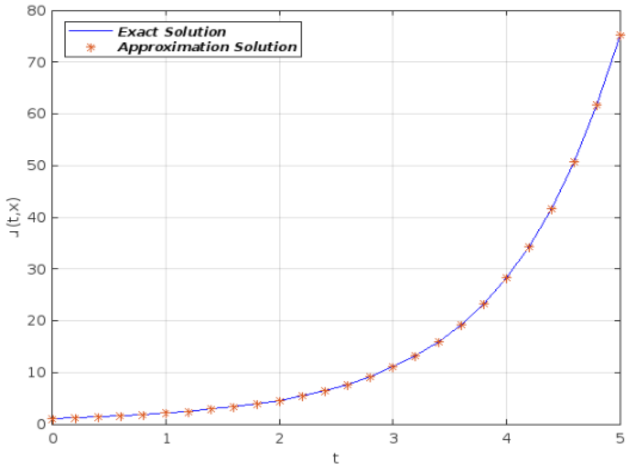

Figure 3

The Graph of eq.(48) and eq.(49) when $n = 20, x = 1$.

Problem VII

The eq. (1) and (2) will be solved with conditions

$$f(x) = x^2, \qquad g(x) = 1, \tag{50}$$

$$\omega(u) = u^2,$$

$$S(x,t) = -t^2 - 4tx + 8tx^2 + 6x^4 \tag{51}$$

The eq. (1) and eq. (2) with conditions (51) and is the inhomogeneous nonlinear wave equation in one dimension, the first iteration is $u_0 = x^2 + t$, since $u_0^2.\frac{\partial^2 u_0}{\partial x^2} + 2u_0 \frac{\partial^2 u_0}{\partial x^2} = S(x,t)$ then according to **Case (5)**, the particular analytical solution to this problem becomes

$$u(x,t) = x^2 + t, \tag{52}$$

Problem VIII

Let the initial conditions are:

$$f(x) = x, \qquad g(x) = 0, \tag{53}$$

and

$$\omega(u) = u^2, \ \ S(x,t) = 2x\cosh(t)\sinh^2(t). \tag{54}$$

The eq. (1) and eq. (2) with conditions (54) and is the inhomogeneous nonlinear wave equation in one dimension, via the iteration equation:

$$u_n = x\left(1 + \frac{1}{2!}t^2 + \frac{1}{4!}t^4 + \frac{1}{6!}t^6 + \frac{1}{8!}t^8 + \cdots + \frac{1}{2n!}t^{2n}\right). \qquad (55)$$

The solution of Problem VIII becomes

$$u(x,t) = x\cosh(t). \qquad (56)$$

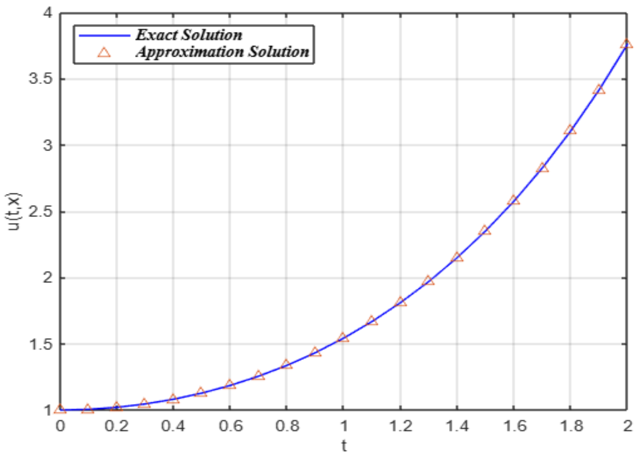

Figure 4

The Graph of eq. (56) and eq. (55) when $n = 20$, $x = 1$.

Problem IX

The eq. (1) and eq. (2) with the conditions

$$f(x) = \sqrt[4]{x}, \quad g(x) = \frac{1}{4\sqrt[4]{x^3}}, \qquad (57)$$

$$\omega(u) = u^3,$$
$$S(x,t) = 2\sqrt[4]{x}e^{t^2} + 4\sqrt[4]{x}t^2e^{t^2}. \qquad (58)$$

The eq. (1) and eq. (2) with conditions (57) and (58) is the inhomogeneous nonlinear wave equation in one dimension. Here is the approximate n^{th} order solution

$$u_n = \sqrt[4]{x}\left(1 + t^2 + \frac{1}{2}t^4 + \frac{1}{6}t^6 + \frac{1}{24}t^8 + \cdots + \frac{1}{n!}t^{2n}\right), \qquad (59)$$

Then the analytical solution of Problem IX

$$u(x,t) = \sqrt[4]{x}e^{t^2}. \qquad (60)$$

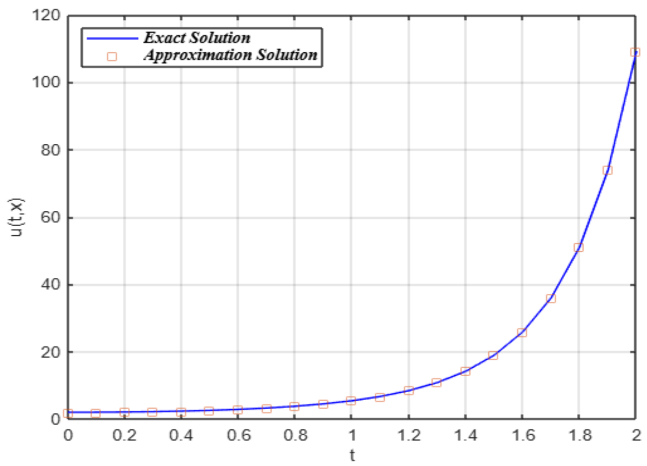

Figure 5
The Graph of eq.(60) and eq.(59) when $n = 20$, $x = 16$.

Problem X

$$f(x) = 0 \ , \quad g(x) = 2(x + 6), \tag{61}$$

$$\omega(u) = x(u - 2x - 12), \quad S(x,t) = (2x + 3)\sin^2(2t). \tag{62}$$

The eq. (1) and eq. (2) with conditions (61) and (62) (47) is the inhomogeneous nonlinear wave equation in one dimension. Here is the approximate n[th] order solution

$$u_n = (x + 3)\left(2t + \frac{1}{3!}(2t)^3 + \frac{1}{5!}(2t)^5 + \frac{1}{7!}(2t)^7 + \frac{1}{9!}(2t)^9 + \frac{1}{11!}(2t)^{11} + \cdots + \frac{1}{(2n+1)!}(2t)^{2n+1}\right) \tag{63}$$

Then, the exact solution of Problem X is:

$$u(x,t) = (x + 3)\sin(2t). \tag{64}$$

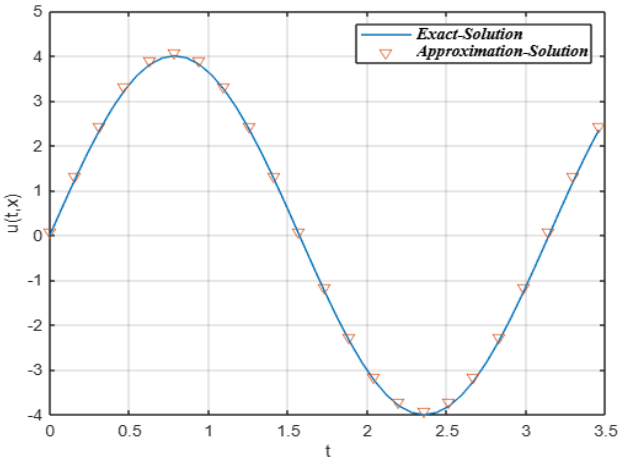

Figure 6
The Graph of eq. (64) and eq. (63) when $n = 20$, $x = -2$.

5. Conclusion

In this study, we discuss the non-linear initial value problem of the wave equations in general form and derive the analytic solution using semi-analytic procedures, specifically the iteration variational method.

Furthermore, we endeavored to provide an extensive number of solutions to this well-known initial value problem. We calculate the solutions for ten homogeneous and inhomogeneous miscellaneous problems.

Although the Taylor series represented the solution for the nonlinear wave propagation problems, this method led us to the solution quickly and easily without requiring us to perform complex analytical steps. We have achieved and tested an excellent level of accuracy in these solutions by comparing them with the exact solution at $n = 20$ see Figure 1, Figure 2, Figure 3, Figure 4 and Figure 5.

Additionally, in certain problems such as Problem I, Problem II and Problem VII, the successive approximations resulted in the exact solution directly after multiple iterations.

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