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Constructing Conflict-Free Schedules via the Affine Plane

$AG(2, 5)^*$

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Abstract

This paper studies a scheduling problem related to organizing twenty-five students into six separate work sessions. In each session, the students are divided into five groups, with five students in each group. One important condition in this problem is that any two students are allowed to work together only once during all sessions. This condition helps ensure fairness and prevents repeated grouping. To solve this problem, the affine plane of order five, $AG(2,5)$, is used as a mathematical model. The affine plane is constructed algebraically from the projective plane $PG(2,5)$ over the finite field F_5 . In the proposed model, students are represented by points, while work groups are represented by lines. The use of parallel classes in $AG(2,5)$ makes it possible to organize the groups without conflicts, so that no student pair is repeated. Although this study focuses on the case of twenty-five students, the results indicate that the combination of finite geometry and companion matrix methods can also be useful for solving similar scheduling and allocation problems.

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بناء جداول خالية من التضارب عبر المستوى التآلفي $AG(2, 5)$

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المستخلص

يدرس هذا البحث مشكلة جدولة تتعلق بتنظيم خمسة وعشرين طالباً ضمن ست جلسات عمل مستقلة. وفي كل جلسة، يتم تقسيم الطلاب إلى خمس مجموعات، بحيث تضم كل مجموعة خمسة طلاب. ويُفرض شرط أساسي في هذه المشكلة، وهو أن يعمل أي طالبان معاً مرة واحدة فقط خلال جميع الجلسات، وذلك من أجل تحقيق العدالة ومنع تكرار المجموعات نفسها. لحل هذه المشكلة، تم استخدام المستوى التآلفي من الرتبة خمسة $AG(2, 5)$ كنموذج رياضي. ويتم بناء هذا المستوى التآلفي بطريقة جبرية انطلاقاً من المستوى الإسقاطي $PG(2, 5)$ على الحقل المنتهي F_5 . ويمثل الطلاب في هذا النموذج النقاط، بينما تمثل مجموعات العمل الخطوط. يساعد استخدام الأصناف المتوازية في المستوى $AG(2, 5)$ على تنظيم المجموعات دون حدوث تعارض، بحيث لا يتكرر أي زوج من الطلاب. وعلى الرغم من أن هذا البحث يركز على حالة خمسة وعشرين طالباً، فإن النتائج تشير إلى أن الجمع بين طرق الهندسة المنتهية ومصفوفة المرافقة يمكن أن يكون مفيداً في حل مشكلات جدولة وتخصيص مشابهة.

الكلمات المفتاحية: بحوث العمليات، الجدولة، الهندسة المنتهية، المستوى التآلفي $AG(2, 5)$ ، مصفوفة المرافقة.

1. Introduction

Operations Research (OR) is a scientific field that is concerned with helping decision-makers solve practical problems in different systems, such as industrial, administrative, and educational systems. (Churchman et al., 1957) stated that Operations Research depends on the use of scientific methods to study operational problems and to assist decision-makers in choosing appropriate solutions. One of the main topics in Operations Research is scheduling, because it is closely related to real-life situations. According to (Taha, 2017), scheduling is the process of assigning limited resources to different activities over time in order to achieve certain objectives while respecting given constraints. Scheduling is widely used in many fields, especially in manufacturing and logistics, as mentioned by (Fleischmann et al., 2001).

Most traditional methods in OR depend on linear programming and statistical techniques. However, other mathematical approaches can also be used to solve scheduling problems. Finite geometry is one of these approaches. (Hirschfeld, 1998) showed that affine and projective planes have useful mathematical properties that allow the construction of balanced combinatorial designs. Based on this idea, (Alabdullah, 2018) used affine planes to solve scheduling problems. In addition, (Mohammed, 1988) and (Al-Zangana & Yahya, 2022) explained that the companion matrix is an effective algebraic tool for constructing finite geometrical structures.

In this paper, the affine plane $AG(2,5)$ is used to solve a scheduling problem involving 25 students, which is a common case in educational institutions (Rajgopal, 2001). The companion matrix method is applied to generate the parallel classes of $AG(2,5)$. These classes are then used to construct a schedule without conflicts, which leads to an efficient and fair allocation of resources.

2. Basic Concepts

2.1 The Projective Plane $PG(2,q)$ (Hirschfeld, 1998)

Let F_q be a finite field, and let $PG(2, q) = (\mathbb{P}_{PG}, \mathbb{L}_{PG})$ be the projective plane over F_q , where $\mathbb{P}_{PG}$ denotes the set of points and $\mathbb{L}_{PG}$ denotes the set of lines. This incidence structure satisfies the following properties:

1. Incidence: For any two distinct points $P_1, P_2 \in P_{PG}$, there exists a unique line $L \in L_{PG}$ such that $P_1, P_2 \in L$.
2. Intersection: For any two distinct lines $L_1, L_2 \in L_{PG}$, there exists a unique point $P \in P_{PG}$ such that $L_1 \cap L_2 = \{P\}$
3. Cardinality: Each line contains exactly $q + 1$ points, each point lies on exactly $q + 1$ lines, and the total number of points and lines is $q^2 + q + 1$.

2.2 The Affine Plane $AG(2, q)$ and its Construction (Hirschfeld, 1998)

The affine plane $AG(2, q)$ is obtained from the projective plane $PG(2, q)$ by removing a fixed line L_∞ , called the line at infinity, along with all points incident with it. Let $AG(2, q) = (P_{AG}, L_{AG})$ denote the resulting incidence structure, where P_{AG} is the set of points and L_{AG} is the set of lines. This structure satisfies the following properties:

1. Incidence: Any two distinct points $P_1, P_2 \in P_{AG}$ lie on a unique line $L \in L_{AG}$.
2. Parallelism: For any line $L \in L_{AG}$ and any point $P \notin L$, there exists exactly one line through P that is parallel to L .
3. Parallel Classes: The lines of $AG(2, q)$ can be partitioned into $q + 1$ parallel classes, each consisting of q mutually disjoint lines that collectively cover all points of the plane.
4. Cardinality: The affine plane contains q^2 points and $q^2 + q$ lines.

3. Results and Discussion

3.1 Problem Statement

This study deals with a scheduling problem related to assigning twenty-five students to six different work sessions ($\Omega_1 - \Omega_6$). In each session, the students are divided into five groups, and each group includes five students. A basic rule is applied in this schedule, which is that any two students are allowed to work together only once during all sessions. This rule helps avoid repeating the same student pairs and ensures that cooperation among students is distributed fairly across the sessions.

3.2 Construction of the Geometric Model

1. Construction of the $PG(2,5)$: Let $f(x) = x^3 - 2x - 2$ be a primitive polynomial over F_5 . The companion matrix $C(f)$ is defined as:

$$C(f) = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 2 & 2 & 0 \end{pmatrix}$$

Generation of Projective Points (P_{PG}): Let $P_1 = [1, 0, 0]$ be the initial point. The set of points $P_{PG} = \{P_1, P_2, \dots, P_{31}\}$ is generated recursively by:

$$P_{i+1} = P_i C(f), \text{ for } i = 1, \dots, 30$$

Table (1): The set of 31 Points of the Projective Plane $PG(2,5)$.

i	P_i	i	P_i	i	P_i	i	P_i	i	P_i
1	[1, 0, 0]	8	[1, 4, 1]	14	[4, 0, 1]	20	[4, 2, 1]	26	[2, 0, 1]
2	[0, 1, 0]	9	[3, 2, 1]	15	[2, 1, 0]	21	[1, 3, 1]	27	[3, 1, 0]
3	[0, 0, 1]	10	[1, 0, 1]	16	[0, 2, 1]	22	[4, 1, 1]	28	[0, 3, 1]
4	[1, 1, 0]	11	[4, 1, 0]	17	[1, 1, 1]	23	[2, 1, 1]	29	[4, 4, 1]
5	[0, 1, 1]	12	[0, 4, 1]	18	[2, 3, 1]	24	[2, 4, 1]	30	[3, 4, 1]
6	[2, 2, 1]	13	[3, 3, 1]	19	[4, 3, 1]	25	[3, 1, 1]	31	[3, 0, 1]
7	[1, 2, 1]								

Generation of Projective Lines (L_{PG}): Let L_1 be the fundamental line defined by points with a zero incidence in the third coordinate ($z = 0$):

$$L_1 = \{t: P_t [x, y, z] \in P_{PG}, z = 0\} = \{1, 2, 4, 11, 15, 27\}$$

The set of lines $L_{PG} = \{L_1, L_2, \dots, L_{31}\}$ is generated by :

$$L_i = \{(t + (i - 1)) \pmod{31}: t \in L_1\}, \text{ for } i = 1, 2, \dots, 31.$$

Table (2): The set of 31 Lines of the Projective Plane

L_i	$\alpha = P_\alpha$	L_i	$\alpha = P_\alpha$	L_i	$\alpha = P_\alpha$
L_1	1, 2, 4, 11, 15, 27	L_{12}	12, 13, 15, 22, 26, 7	L_{22}	22, 23, 25, 1, 5, 17
L_2	2, 3, 5, 12, 16, 28	L_{13}	13, 14, 16, 23, 27, 8	L_{23}	23, 24, 26, 2, 6, 18
L_3	3, 4, 6, 13, 17, 29	L_{14}	14, 15, 17, 24, 28, 9	L_{24}	24, 25, 27, 3, 7, 19
L_4	4, 5, 7, 14, 18, 30	L_{15}	15, 16, 18, 25, 29, 10	L_{25}	25, 26, 28, 4, 8, 20
L_5	5, 6, 8, 15, 19, 31	L_{16}	16, 17, 19, 26, 30, 11	L_{26}	26, 27, 29, 5, 9, 21
L_6	6, 7, 9, 16, 20, 1	L_{17}	17, 18, 20, 27, 31, 12	L_{27}	27, 28, 30, 6, 10, 22

$\mathbb{L}_7$	7, 8, 10, 17, 21, 2	$\mathbb{L}_{18}$	18, 19, 21, 28, 1, 13	$\mathbb{L}_{28}$	28, 29, 31, 7, 11, 23
$\mathbb{L}_8$	8, 9, 11, 18, 22, 3	$\mathbb{L}_{19}$	19, 20, 22, 29, 2, 14	$\mathbb{L}_{29}$	29, 30, 1, 8, 12, 24
$\mathbb{L}_9$	9, 10, 12, 19, 23, 4	$\mathbb{L}_{20}$	20, 21, 23, 30, 3, 15	$\mathbb{L}_{30}$	30, 31, 2, 9, 13, 25
$\mathbb{L}_{10}$	10, 11, 13, 20, 24, 5	$\mathbb{L}_{21}$	21, 22, 24, 31, 4, 16	$\mathbb{L}_{31}$	31, 1, 3, 10, 14, 26
$\mathbb{L}_{11}$	11, 12, 14, 21, 25, 6				

2. Derivation of the Affine Plane $AG(2,5)$: The Affine Plane is derived by fixing the line at infinity $\mathbb{L}_\infty = \mathbb{L}_1$.

Affine Points (The Students): The set of affine points $\mathbb{P}_{AG}$ is obtained by the set difference between projective points and the points incident to $\mathbb{L}_1$. These points mathematically represent the set of Students $\mathbb{S} : \mathbb{P}_{AG} = \mathbb{P}_{PG} \setminus \{ \mathbb{P} \in \mathbb{P}_{PG} : \mathbb{P} \in \mathbb{L}_\infty \}$

We define the mapping: $\mathbb{S} = \{ \mathbb{S}_1, \mathbb{S}_2, \dots, \mathbb{S}_{25} \} \cong \mathbb{P}_{AG}$ where each point $\mathbb{P}_k \in \mathbb{P}_{AG}$ corresponds to a unique student $\mathbb{S}_k$.

Table (3): The 25 Points of the Affine Plane representing Students

$\mathbb{S}_v$	α	$\mathbb{P}_\alpha$	$\mathbb{S}_v$	α	$\mathbb{P}_\alpha$	$\mathbb{S}_v$	α	$\mathbb{P}_\alpha$
$\mathbb{S}_1$	3	[0, 0, 1]	$\mathbb{S}_{10}$	14	[4, 0, 1]	$\mathbb{S}_{19}$	24	[2, 4, 1]
$\mathbb{S}_2$	5	[0, 1, 1]	$\mathbb{S}_{11}$	16	[0, 2, 1]	$\mathbb{S}_{20}$	26	[2, 0, 1]
$\mathbb{S}_3$	6	[2, 2, 1]	$\mathbb{S}_{12}$	17	[1, 1, 1]	$\mathbb{S}_{21}$	27	[3, 1, 0]
$\mathbb{S}_4$	7	[1, 2, 1]	$\mathbb{S}_{13}$	18	[2, 3, 1]	$\mathbb{S}_{22}$	28	[0, 3, 1]
$\mathbb{S}_5$	8	[1, 4, 1]	$\mathbb{S}_{14}$	19	[4, 3, 1]	$\mathbb{S}_{23}$	29	[4, 4, 1]
$\mathbb{S}_6$	9	[3, 2, 1]	$\mathbb{S}_{15}$	20	[4, 2, 1]	$\mathbb{S}_{24}$	30	[3, 4, 1]
$\mathbb{S}_7$	10	[1, 0, 1]	$\mathbb{S}_{16}$	21	[1, 3, 1]	$\mathbb{S}_{25}$	31	[3, 0, 1]
$\mathbb{S}_8$	12	[0, 4, 1]	$\mathbb{S}_{17}$	22	[4, 1, 1]			
$\mathbb{S}_9$	13	[3, 3, 1]	$\mathbb{S}_{18}$	23	[2, 1, 1]			

Affine Lines (The Groups): The set of affine lines $\mathbb{L}_{AG} = \{ \mathbb{L}_1, \mathbb{L}_2, \dots, \mathbb{L}_{30} \}$ is obtained by excluding the line at infinity and its points from the set of projective lines. These lines mathematically represent the set of Groups G :

$$\mathbb{L}_{AG} = \{ \mathbb{L}_\sigma : \mathbb{L}_\sigma = \mathbb{L}_{\sigma+1} \setminus (\mathbb{L}_{\sigma+1} \cap \mathbb{L}_1), \mathbb{L}_{\sigma+1} \in \mathbb{L}_{PG} \setminus \{ \mathbb{L}_1 \} \}$$

We define the mapping: $G = \{ G_1, G_2, \dots, G_{30} \} \cong \mathbb{L}_{AG}$ where each line $\mathbb{L}_k \in \mathbb{L}_{AG}$ corresponds to a unique working group G_k .

Table (4): The 30 Lines of the Affine Plane representing Groups.

$\mathbb{L}_\sigma$	$\alpha = \mathbb{P}_\alpha$	$\mathbb{L}_\sigma$	$\alpha = \mathbb{P}_\alpha$	$\mathbb{L}_\sigma$	$\alpha = \mathbb{P}_\alpha$
$\mathbb{L}_1$	3, 5, 12, 16, 28	$\mathbb{L}_{11}$	12, 13, 22, 26, 7	$\mathbb{L}_{21}$	22, 23, 25, 5, 17
$\mathbb{L}_2$	3, 6, 13, 17, 29	$\mathbb{L}_{12}$	13, 14, 16, 23, 8	$\mathbb{L}_{22}$	23, 24, 26, 6, 18
$\mathbb{L}_3$	5, 7, 14, 18, 30	$\mathbb{L}_{13}$	14, 17, 24, 28, 9	$\mathbb{L}_{23}$	24, 25, 3, 7, 19
$\mathbb{L}_4$	5, 6, 8, 19, 31	$\mathbb{L}_{14}$	16, 18, 25, 29, 10	$\mathbb{L}_{24}$	25, 26, 28, 8, 20
$\mathbb{L}_5$	6, 7, 9, 16, 20	$\mathbb{L}_{15}$	16, 17, 19, 26, 30	$\mathbb{L}_{25}$	26, 29, 5, 9, 21
$\mathbb{L}_6$	7, 8, 10, 17, 21	$\mathbb{L}_{16}$	17, 18, 20, 31, 12	$\mathbb{L}_{26}$	28, 30, 6, 10, 22
$\mathbb{L}_7$	8, 9, 18, 22, 3	$\mathbb{L}_{17}$	18, 19, 21, 28, 13	$\mathbb{L}_{27}$	28, 29, 31, 7, 23
$\mathbb{L}_8$	9, 10, 12, 19, 23	$\mathbb{L}_{18}$	19, 20, 22, 29, 14	$\mathbb{L}_{28}$	29, 30, 8, 12, 24
$\mathbb{L}_9$	10, 13, 20, 24, 5	$\mathbb{L}_{19}$	20, 21, 23, 30, 3	$\mathbb{L}_{29}$	30, 31, 9, 13, 25
$\mathbb{L}_{10}$	12, 14, 21, 25, 6	$\mathbb{L}_{20}$	21, 22, 24, 31, 16	$\mathbb{L}_{30}$	31, 3, 10, 14, 26

3.3 Scheduling Implementation

The practical application of the schedule depends on connecting the structure of the affine plane $AG(2,5)$ with the required organizational rules. In this model, the 25 points of the affine plane are used to represent the students ($\mathbb{S}_v$), while the 30 affine lines represent the working groups (G_σ).

Formation of Work Sessions: The six work sessions ($\Omega_1 - \Omega_6$) are constructed using the six points lying on a selected reference line, denoted by ($\mathbb{L}_1$), which is treated as the line at infinity. Each work session corresponds to one parallel class ($\mathcal{C}$) of affine lines. These lines are obtained by removing one point at infinity from the reference line ($\mathbb{L}_1$). This relationship can be expressed as:

$$\Omega_a = \mathcal{C}_a = \{ \mathbb{L}_a : \mathbb{L}_a = \mathbb{L}_{a+1} \setminus P_t, P_t \in \mathbb{L}_1 \}.$$

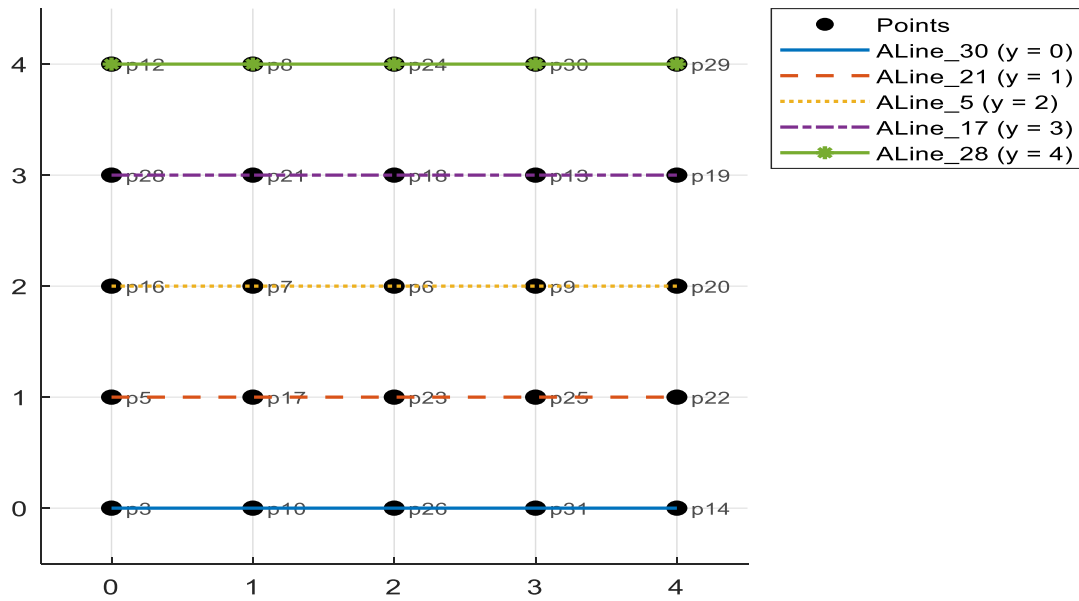
Within each work session, the groups are described by simple linear equations of the form $mx + y = c$, or $x = c$ in the case of vertical lines. The value of m represents the slope related to the removed point at infinity, while the value $c \in F_5$ specifies the individual group inside the session.

Validation of the Schedule: The validity of the proposed schedule is based on basic properties of the affine plane. Since any two distinct points lie on exactly one affine line, it follows that any pair of students can appear together in the same group only once during the entire schedule. This property guarantees that the schedule does not contain repeated groupings or conflicts. The final

distribution of students and its geometric representation are illustrated in Table (5) and Figures (1–6).

Table (5): Allocation of Twenty-Five Students Across Six Work Sessions.

$C_a = \Omega_a = \{\mathbb{L}_a = \mathbb{L}_{a+1} \setminus \mathbb{P}_t\}$	$\mathbb{L}_a$	$\alpha = \mathbb{P}_\alpha$	Eq_a
$C_1 = \{\mathbb{L}_a = \mathbb{L}_{a+1} \setminus \mathbb{P}_1\}$ $C_1 = \Omega_1$	$\mathbb{L}_5$	6, 7, 9, 16, 20	$y = 2$
	$\mathbb{L}_{17}$	18, 19, 21, 28, 13	$y = 3$
	$\mathbb{L}_{21}$	22, 23, 25, 5, 17	$y = 1$
	$\mathbb{L}_{28}$	29, 30, 8, 12, 24	$y = 4$
	$\mathbb{L}_{30}$	31, 3, 10, 14, 26	$y = 0$
$C_2 = \{\mathbb{L}_a = \mathbb{L}_{a+1} \setminus \mathbb{P}_2\}$ $C_2 = \Omega_2$	$\mathbb{L}_1$	3, 5, 12, 16, 28	$x = 0$
	$\mathbb{L}_6$	7, 8, 10, 17, 21	$x = 1$
	$\mathbb{L}_{18}$	19, 20, 22, 29, 14	$x = 4$
	$\mathbb{L}_{22}$	23, 24, 26, 6, 18	$x = 2$
	$\mathbb{L}_{29}$	30, 31, 9, 13, 25	$x = 3$
$C_3 = \{\mathbb{L}_a = \mathbb{L}_{a+1} \setminus \mathbb{P}_{11}\}$ $C_3 = \Omega_3$	$\mathbb{L}_7$	8, 9, 18, 22, 3	$x + y = 0$
	$\mathbb{L}_9$	10, 13, 20, 24, 5	$x + y = 1$
	$\mathbb{L}_{10}$	12, 14, 21, 25, 6	$x + y = 4$
	$\mathbb{L}_{15}$	16, 17, 19, 26, 30	$x + y = 2$
	$\mathbb{L}_{27}$	28, 29, 31, 7, 23	$x + y = 3$
$C_4 = \{\mathbb{L}_a = \mathbb{L}_{a+1} \setminus \mathbb{P}_{15}\}$ $C_4 = \Omega_4$	$\mathbb{L}_4$	5, 6, 8, 19, 31	$2x + y = 1$
	$\mathbb{L}_{11}$	12, 13, 22, 26, 7	$2x + y = 4$
	$\mathbb{L}_{13}$	14, 17, 24, 28, 9	$2x + y = 3$
	$\mathbb{L}_{14}$	16, 18, 25, 29, 10	$2x + y = 2$
	$\mathbb{L}_{19}$	20, 21, 23, 30, 3	$2x + y = 0$
$C_5 = \{\mathbb{L}_a = \mathbb{L}_{a+1} \setminus \mathbb{P}_{27}\}$ $C_5 = \Omega_5$	$\mathbb{L}_{12}$	13, 14, 16, 23, 8	$3x + y = 2$
	$\mathbb{L}_{16}$	17, 18, 20, 31, 12	$3x + y = 4$
	$\mathbb{L}_{23}$	24, 25, 3, 7, 19	$3x + y = 0$
	$\mathbb{L}_{25}$	26, 29, 5, 9, 21	$3x + y = 1$
	$\mathbb{L}_{26}$	28, 30, 6, 10, 22	$3x + y = 3$
$C_6 = \{\mathbb{L}_a = \mathbb{L}_{a+1} \setminus \mathbb{P}_4\}$ $C_6 = \Omega_6$	$\mathbb{L}_2$	3, 6, 13, 17, 29	$4x + y = 0$
	$\mathbb{L}_3$	5, 7, 14, 18, 30	$4x + y = 1$
	$\mathbb{L}_8$	9, 10, 12, 19, 23	$4x + y = 4$
	$\mathbb{L}_{20}$	21, 22, 24, 31, 16	$4x + y = 2$
	$\mathbb{L}_{24}$	25, 26, 28, 8, 20	$4x + y = 3$



Figures (1): Structure of Work Session (Ω_1) Based on Parallel Class (C_1) .

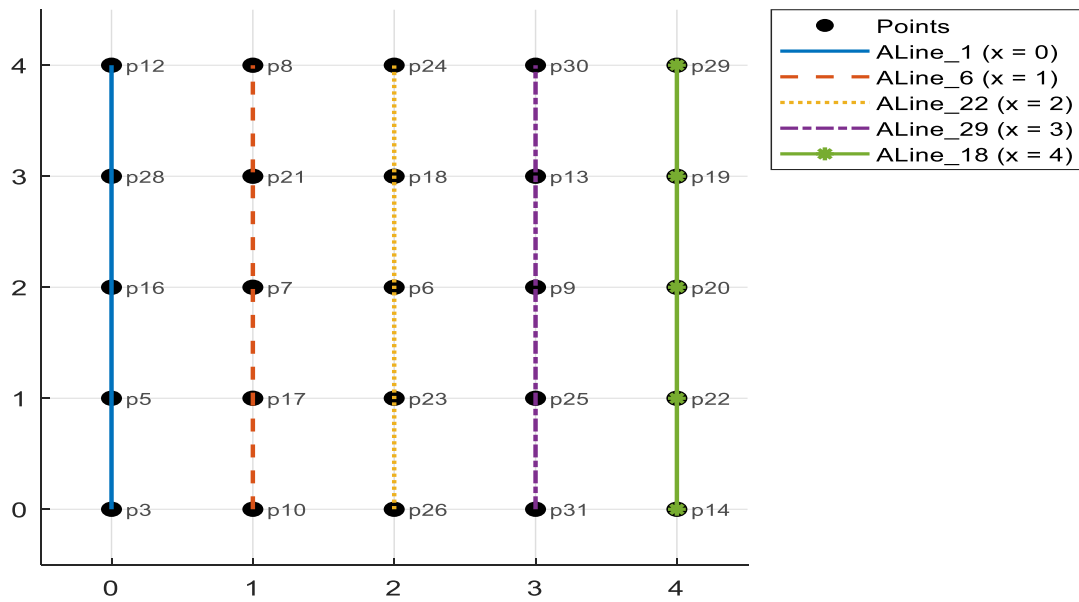


Figure (2): Structure of Work Session (Ω_2) Based on Parallel Class (C_2) .

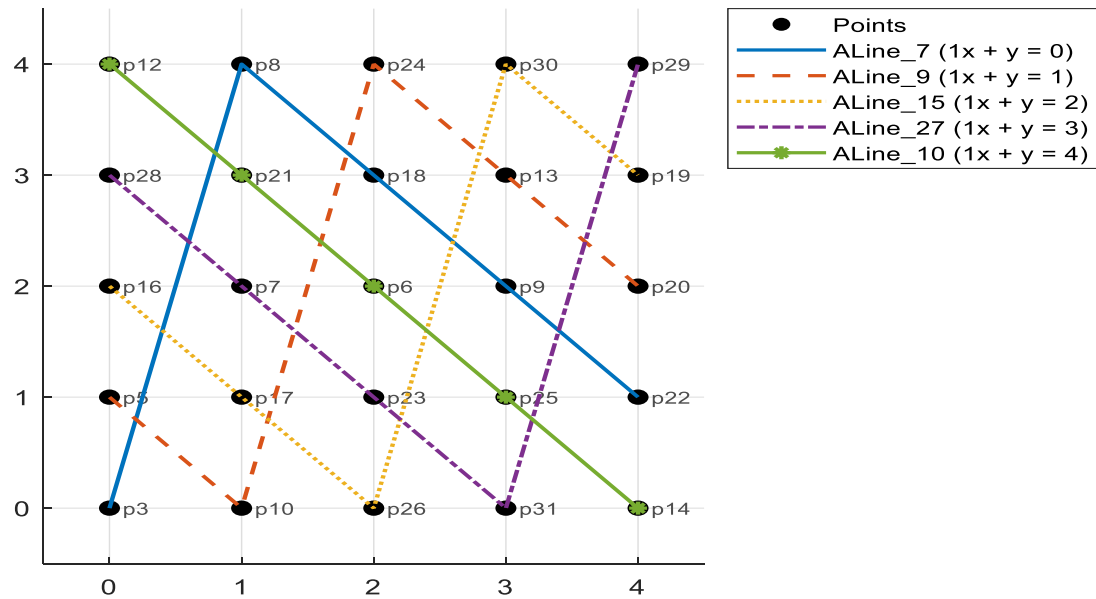


Figure (3): Structure of Work Session (Ω_3) Based on Parallel Class (C_3)

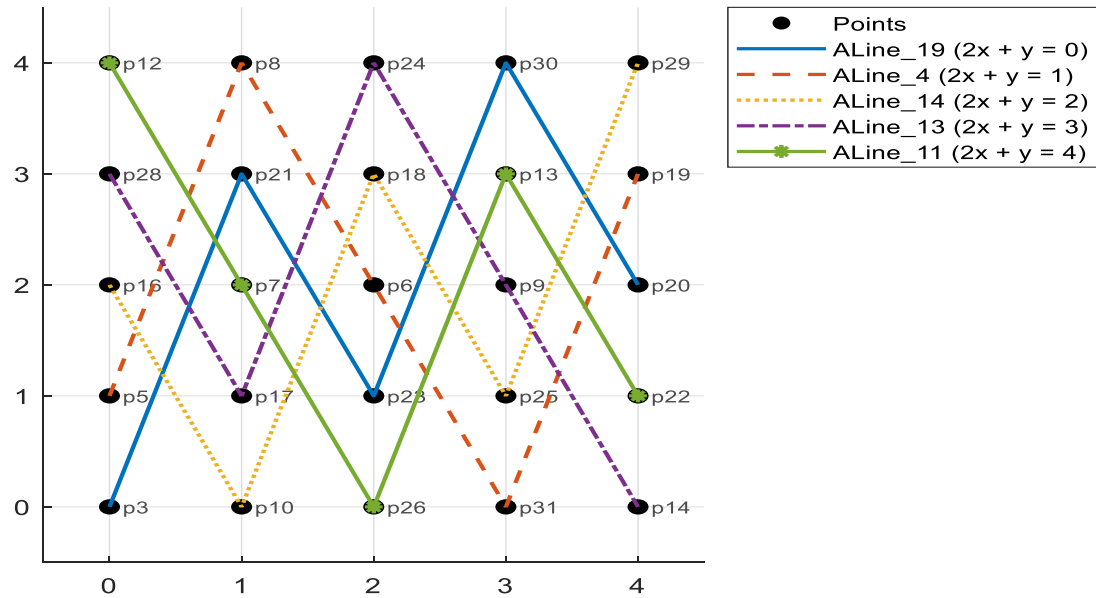


Figure (4): Structure of Work Session (Ω_4) Based on Parallel Class (C_4).

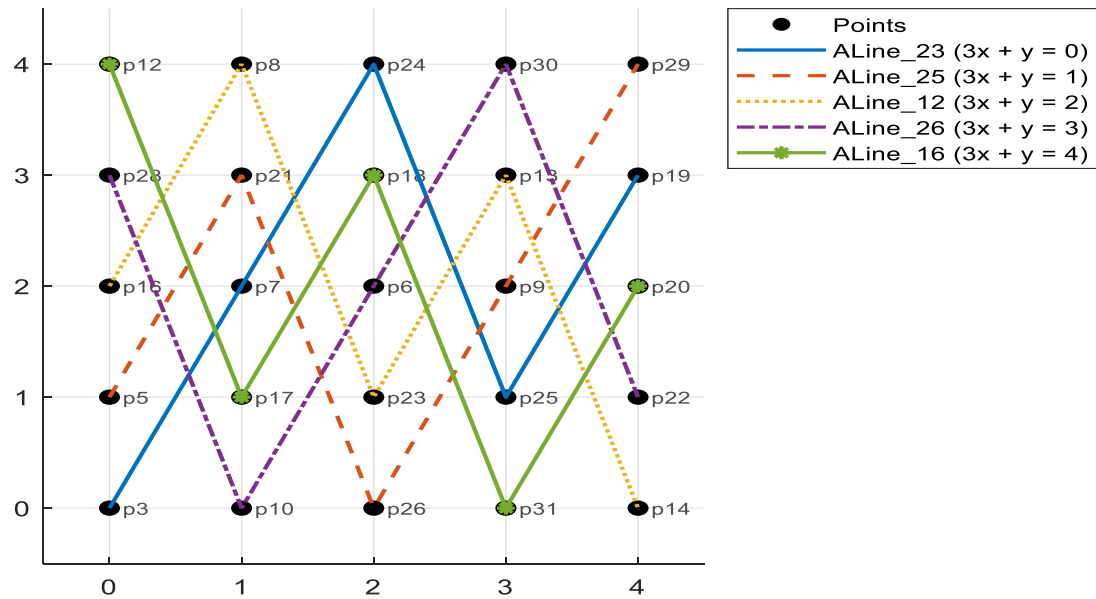


Figure (5): Structure of Work Session (Ω_5) Based on Parallel Class (C_5)

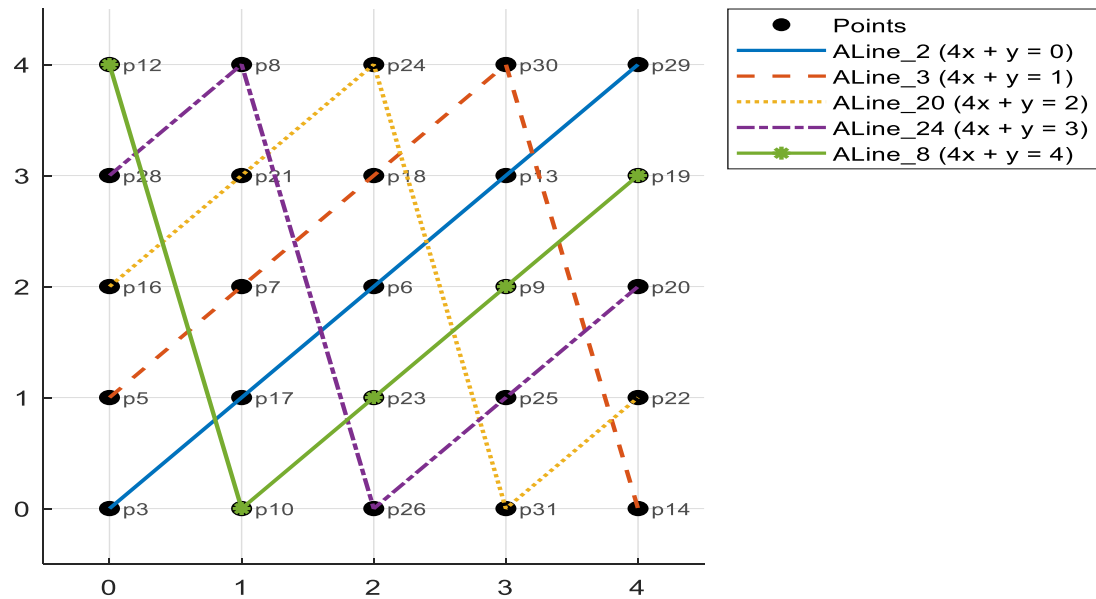


Figure (6): Structure of Work Session (Ω_6) Based on Parallel Class (C_6)

4. Conclusions

This research shows that the Affine Plane $AG(2,5)$ provides a systematic and well-defined mathematical method for solving scheduling problems, and that it is more effective than informal trial-and-error approaches. By employing the companion matrix together with the basic axioms of finite geometry, the proposed schedule is constructed in a way that naturally avoids conflicts, since the non-repetition condition is satisfied within the structure of the model itself. Although the current study focuses on the case $N = q^2 = 5^2 = 25$, the logic of the companion matrix construction can be generalized to affine planes $AG(2, q)$ of any prime power order q . This makes the proposed method suitable for a broader range of allocation and organizational problems.

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